Interpretation of single-well tracer tests using fractional-flow dimensions. Part 2: A case study

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Abstract Generalized equations using fractional-flow dimensions were derived to estimate the Darcy and groundwater-flow velocities obtained from the point-dilution and the single-well injection-withdrawal field tests. Flow velocities can only be estimated from single-well tests if the kinematic porosity or the hydraulic conductivity and hydraulic gradient are known a priori. A pumping test performed on the boreholes will yield an estimate of the fractional-flow dimension and the extent of the flow region by applying the generalized radial flow (GRF) model of Barker [Barker JA (1988) A generalized radial flow model for hydraulic tests in fractured rock. Water Resour Res 24(10):1796–1804]. These parameters are used in the generalised equations for the single-well tracer tests to estimate the flow-through area and, therewith, the Darcy or flow velocity.

The generalized procedure, described in detail in Part 1 of this paper, is applied to two boreholes on the Campus Test Site located at the University of the Free State, Bloemfontein, South Africa, and it is shown that the two independent tests (i.e. the point-dilution and the single-well injection-withdrawal tests) yield similar estimates of the natural-seepage velocity in the aquifer. The estimated natural-flow velocity obtained by using fractional dimensions is about two times higher than the velocity estimated by using the standard method (i.e. flow dimension *n*=2, flow thickness equal to length of the sealed-off section of the borehole).

Résumé Des équations généralisées utilisant des dimensions d'écoulement fractionnaire ont été dérivées afin d'estimer les vitesses de Darcy et d'écoulement souterrain obtenues à partir d'essais de terrain de dilution ponctuelle et d'injection-pompage sur puits unique. Les vitesses d'écoulement peuvent être estimées uniquement à partir d'essais de puits unique si la porosité cinématique ou la conductivité hydraulique et le gradient hydraulique sont connus a priori. Un essai de pompage réalisé dans les forages fournira une estimation de la dimension de l'écoulement fractionnaire et de l'étendue de la région d'écoulement en appliquant le modèle d'écoulement radial généralisé (GRF) de Barker. Ces paramètres sont utilisés dans les équations généralisées pour des essais de traçage en puits unique pour estimer la zone traversée par l'écoulement et, en outre, la vitesse de Darcy ou la vitesse de l'écoulement. Le modèle d'écoulement radial généralisé (GRF) de Barker [Barker JA (1988) A generalized radial flow model for hydraulic tests in fractured rock. Water Resour Res 24(10):1796–1804] est utilisé pour estimer la dimension de l'écoulment du fait de sa large gamme.

La procédure généralisée, décrite en détail dans la partie 1 de cet article, est appliquée à deux forages du site expérimental du campus de l'Université de l'État Libre (Bloemfontein, Afrique du Sud). On montre que les deux essais indépendants (c'est-à-dire l'essai de dilution ponctuelle et celui d'injection-pompage sur puits unique) donnent des estimations comparables de la vitesse d'écoulement naturel dans l'aquifère. La vitesse estimée de l'écoulement naturel obtenue à partir des dimensions fractionnaires est environ 2 fois plus élevée que la vitesse estimée par la méthode standard (c'est-à-dire une dimension d'écoulement *n*=2 et une épaisseur de l'écoulement égale à la longueur de la section tubée du forage).

Resumen Se ha derivado las ecuaciones generalizadas que permiten estimar las velocidades de Darcy y las de flujo por medio de dimensiones fraccionales. Las medidas de campo han sido obtenidas mediante ensayos de dilución puntual y de inyección y extracción en sondeo único. Las velocidades de flujo sólo pueden ser estimadas a partir de ensayos en sondeo único si se conoce a priori la porosidad cinemática o la conductividad hidráulica y el gradiente hidráulico. La aplicación del modelo de Barker de flujo radial generalizado (FRG) [Barker JA (1988) A generalized radial flow model for hydraulic tests in fractured rock. Water Resour Res 24(10):1796–1804] a un ensayo de bombeo proporciona una estimación de la dimensión fraccional de flujo y del alcance de la región de flujo. Estos parámetros son utilizados en las ecuaciones generalizadas para ensayos de

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trazadores en sondeo único con el fin de estimar el área de flujo y, de aquí, la velocidad de Darcy o de flujo.

Se aplica el procedimiento general, descrito con detalle en la parte 1 de este artículo, a dos sondeos del Campus Test Site, situado en la Universidad de Free State (Bloemfontein, Sudáfrica). Se muestra que los dos ensayos independientes (de dilución puntual, y de inyección y extracción en sondeo único) proporcionan esimaciones similares de la velocidad real natural en el acuífero. La velocidad de flujo natural estimada con dimensiones fraccionales es del orden de 2 veces mayor que la velocidad estimada según el método convencional (es decir, dimensión de flujo igual a 2 y espesor de flujo igual al tramo filtrante del sondeo).

Keywords Fractional-flow dimension · Fractured aquifer · Injection-withdrawal test · Point-dilution test · Tracer

Notation

Refer to 'Interpretation of single-well tracer tests using fractional-flow dimensions. Part 1: Theory and mathematical models', for a complete list of notations used in this investigation.

Introduction

In Part 1 of this paper the generalized equations for analysing tracer tests, using fractional-flow dimensions, were derived and discussed in detail. In this case study the application of these equations is tested on two single-well tracer tests. The equations used for this case study are summarized below.

The generalized Theis equation for flow in a fractured aquifer, which is applicable in this specific case, is written as (Barker 1988):

$$s(r,t) = \frac{Qr^{2N}}{4\pi^{1-N}K_f b^{3-n}}\Gamma(-N,u) \tag{1}$$

where, $u=r^2S_{sf}/4K_ft$; b=extent of flow region (thickness of flow region in case where n=2); N=1–n/2; n = dimension of flow; K_f=K-value of the fracture system; S_{sf}= specific storage (1/m); $\Gamma(-N,u)$= incomplete gamma function; $\Gamma(0, u)=W(u)$= Theis function; and r= the distance along the flow path (mean length of flow paths between the abstraction and observation boreholes).

The Darcy velocity is computed from a dilution test as (Drost and Neumaier 1974):

$$q = -\frac{W}{\alpha At}\ln\left(\frac{C}{C_0}\right) \tag{2}$$

where W= the volume of fluid contained in the test section; A= cross-sectional area normal to the direction of flow; C_0= tracer concentration at t=0; C= tracer concentration at time =t; α= borehole-distortion factor (between 0.5 and 4; =2 for an open well); and t = time where concentration is equal to C (note that $q\alpha$=v*, where v*= apparent velocity inside the well).

In practice, either the radial-flow solution or the parallel-plate model is used to estimate the cross sectional area A (Novakowski 1992; Novakowski et al. 1995):

$$A=\pi r_w d \text{ (for the radial} - \text{flow model)} \tag{3}$$

where r_w= well radius; d= length of the tested section in the borehole, and

$$A=\pi r_w(2b) \text{ for the parallel} - \text{plate model} \tag{4}$$

where 2b= equivalent aperture of the fractured rock.

For n-dimensional flow, the cross-sectional area is given by (half the borehole circumference is used):

$$A = \frac{r_w^{n-1}\pi^{\frac{n}{2}}b^{3-n}}{\Gamma\left(\frac{n}{2}\right)} \tag{5}$$

The single-well injection-withdrawal test (also known as the drift and pump-back test) was first described by Borowczyk et al. (1966) and also by Leap and Kaplan (1988). Groundwater-flow velocity is then calculated, based on the amount of pumping needed to recover the tracer, by the following equation (Leap and Kaplan 1988):

$$v = \frac{\sqrt{Qt_p/\pi\varepsilon D}}{t_d} \tag{6}$$

where v= seepage velocity (m/day); Q = pumping rate during recovery of tracer (m^3/day); t_p= time elapsed from start of pumping until the centre of mass of the tracer is recovered (d); ε= kinematic porosity; D= aquifer thickness (m); and t_d= time elapsed from the injection of tracer until the centre of mass of the tracer is recovered (d).

Equation (6) is applicable in the case where a flow dimension n=2 exists. It is also possible to estimate the seepage velocity in a more general way by using fractional-flow dimensions. Then the flow velocity is computed as:

$$v = \frac{(Qt_p/\varepsilon b^{3-n}\beta_n)^{\frac{1}{n}}}{t_d} \tag{7}$$

where

$$\beta_n = \frac{\pi^{\frac{n}{2}}}{{}^{n}/_{2}\Gamma\left({}^{n}/_{2}\right)}$$

If n=2 (meaning radial-flow field), Eq. (7) reduces to Eq. (6).

The approximate solution for converging radial flow with a pulse injection is given by (Sauty 1980):

$$c(r,t) = \frac{\Delta M}{2Q\sqrt{\pi\alpha_L vt^3}}\exp\left[-\frac{(r-vt)^2}{4D_Lt}\right] \tag{8}$$

where ΔM= injected mass of tracer per unit section [mass (kg)/thickness (m)]; α_L= longitudinal dispersivity (m); D_L= longitudinal-dispersion coefficient (m^2/s); $D_L=\alpha_L\times v$; $v=v_f$, groundwater velocity under forced gradient (m/s); Q= pumping rate of the well (m^3/s); r= radial distance (m) between the two boreholes.

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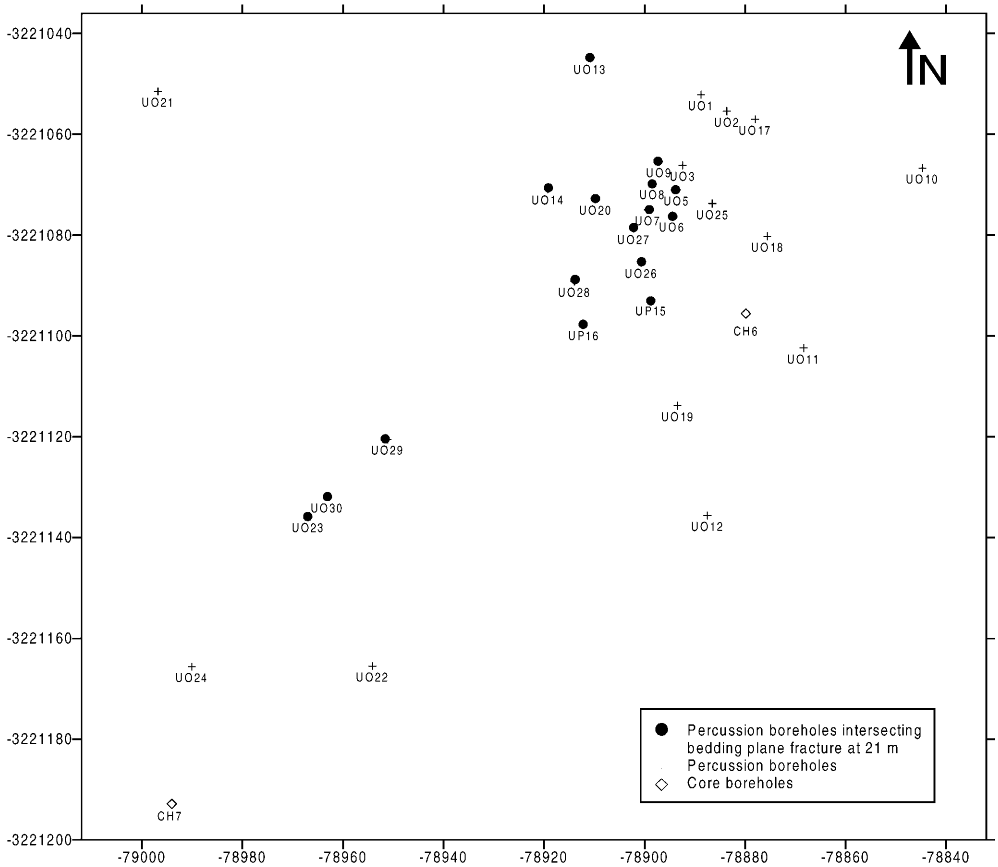

Fig. 1 Borehole layout on the Campus Site. Coordinate origin (vertical axis) –3221036 (m), (horizontal axis) –78832 (m)

From Eq. (8), the velocity under forced gradient v_f could be estimated and also the kinematic porosity from the equation:

$$v = \frac{Q}{\varepsilon A} \tag{9}$$

where A is the through-flow area. According to Eq. (5), the area A for a flow dimension n is given as:

$$A = \frac{2\pi^{\frac{n}{2}} r^{n-1} b^{3-n}}{\Gamma\left(\frac{n}{2}\right)} \tag{10}$$

If n=2, Eq. (10) reduces to $A=2\pi rb$ (through-flow area of a cylinder in which b = thickness of cylinder).

If the point-dilution test is used as the source in the injection borehole, the Darcy velocity q_f can be estimated. From Eq. (8) the groundwater velocity under forced gradient is obtained by fitting Eq. (8) to the data of the breakthrough curve. The relation $v_f=q_f/\varepsilon$ can then be used to estimate the kinematic porosity and this value could be compared with the kinematic porosity estimated with Eq. (9).

The kinematic porosity, estimated from the radial-convergent test, can then be used to estimate the natural-flow velocity from the single-well test.

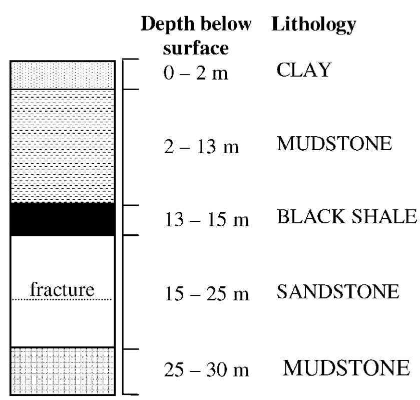

Fig. 2 Lithological log of borehole UO26

Case Study – Campus Test Site

Geology of the Site

The Campus Test Site at the University of the Free State was originally intended as a test site for postgraduate students and covers an area of approximately 180× 192 m. To date, 30 percussion and seven core-boreholes were drilled (Fig. 1). Two projects sponsored by the Water Research Commission of South Africa also used this site for research on (1) Karoo Aquifers (Botha et al. 1998) and (2) tracer tests in fractured aquifers (Van Wyk et al. 2000). The Campus Test Site is underlain by a se-

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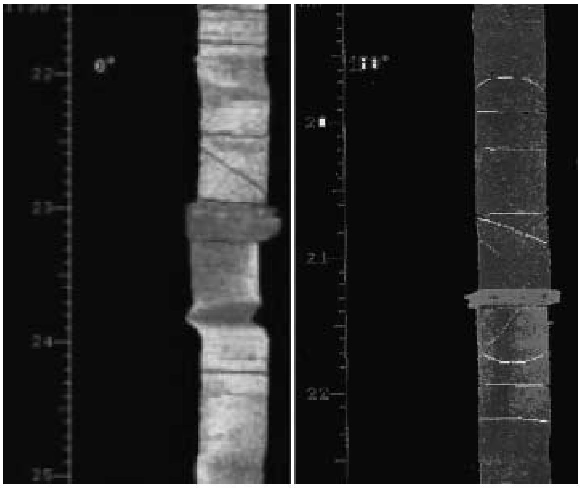

Fig. 3 Acoustic scan of borehole UO5 and UO23 at a depth of 20–25 m below the surface

ries of mudstones and sandstones from the Adelaide Subgroup of the Beaufort Group of formations in the Karoo Sequence. Mapping of geological outcrops around the Campus Site reveals the existence of extensional fractures (mode I) and shearing fractures (mode II). The dominant type of fractures recognised in the sediments includes subhorizontal bedding-parallel fractures and orthogonal and diagonal fractures with dominant north-west, north-east and east–west trends.

Five of the seven core-boreholes were drilled vertically and two at an angle of 45°. The two northeasterly-trending fractures detected in core-boreholes CH6 and CH7 are the only subvertical structures intersected during the core drilling and both were calcified. The strata underlying the Campus Site (Fig. 2) can be subdivided into five different, easily recognizable rock units, each characterized by a unique assemblage of rock types and structures. These lithological units may be subdivided into different lithofacies. The vertical lithofacies represent vertical accretion of deposits in floodplains (mudstone and siltstone facies), shallow lakes (rhythmite facies) and channels (sandstone facies). A major feature of the core samples is the large number of bedding-parallel fractures whose frequency decreases downward from the upper, more weathered zone, as thicker and more competent units are encountered. The bedding-plane fractures in the upper, more weathered part are often transected by a large number of orthogonal, oblique and diagonal fractures. These fractures clearly represent secondary fracturing of the rock mass caused by the post-lithification process.

The mode I fracture is the most significant fracture on the Campus Site and all boreholes with high yields (11 have yields in excess of 3 l/s) intersected this bedding-plane fracture. The yields of the other 19 percussion boreholes are less than 0.6 l/s because the mode I fracture was not intersected during drilling. It is clear from both the acoustic scan and borehole video image of borehole UO23 (Figs. 3 and 4), that the mode I fracture, which is situated at about 21 m below the surface, consists of a fracture zone with a thickness that varies between 100–200 mm (i.e. the fracture zone has developed

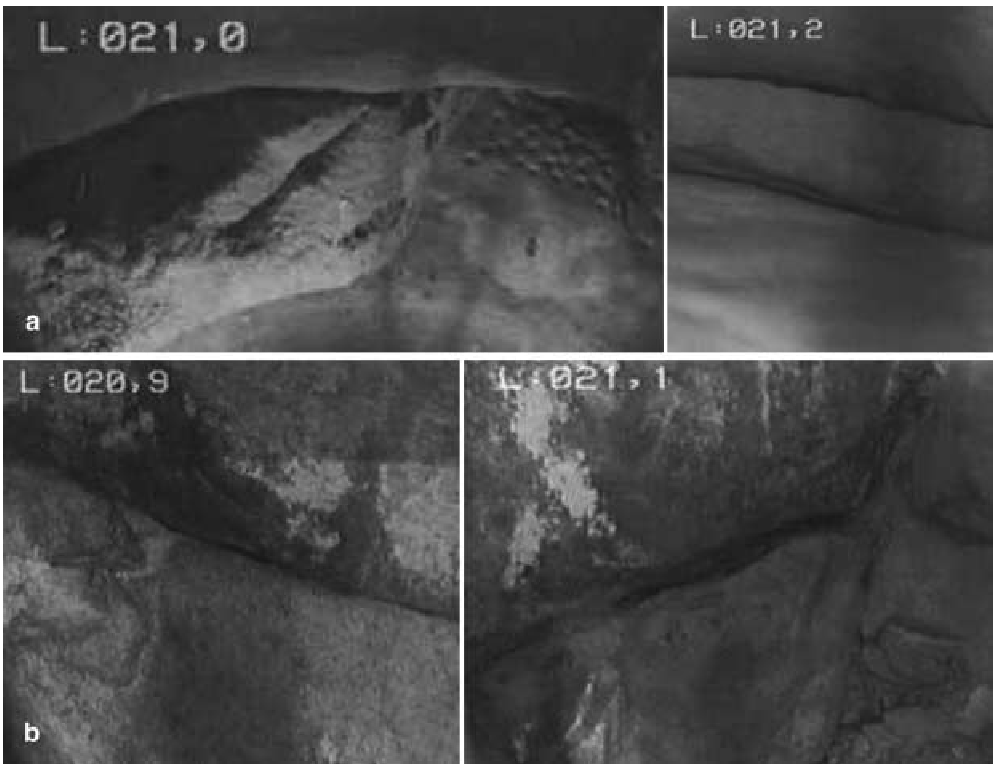

Fig. 4 **a** Borehole video image of the fracture in borehole UO23 at a depth of 21.0–21.2 m below the surface. **b** Borehole video image of the fracture zone in borehole UO5 showing a fracture-zone thickness of about 200 mm

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as consequence of the weathering of the rock between two bedding-plane fractures that were close together).

A very dominant black-shale layer at about 13 m below the surface forms an aquitard between the top mudstone and the underlying sandstone layer (which, thus, could be viewed as semi-confined). There are three aquifers present on the site. A phreatic aquifer at the top occurs within the upper mudstone layers. This aquifer is separated from the middle (main) aquifer, a sandstone layer between 8–10 m thick, by the black layer of carbonaceous shale with a thickness of 0.5–4 m. The bottom aquifer occurs in the mudstone layers (more than 100 m thick) that underlie the sandstone unit (Botha et al. 1998).

Hydraulic Tests

During the project conducted by Botha et al. (1998), a large number of hydraulic tests were performed on open boreholes (diameter 165 mm) including constant-rate withdrawal tests, double-packer and cross-borehole packer tests, as well as slug tests.

A generalized flow model was applied to two constant-rate tests (on boreholes UO5 and UO26, where tracer tests were also conducted).

UO5 test

During this first test, water was abstracted from borehole UO5 at a rate of 1.25 l/s and water levels were measured in boreholes UO6 (distance 6 m), UP15 (distance 22 m) and UP16 (distance 32 m), which intersected the same bedding plane fracture that was penetrated by borehole UO5. Figure 5 shows the measured drawdown values of the test together with the drawdown values measured in borehole UO10 (distance 48 m) in which the mode I fracture does not occur. A very interesting feature is that the water-level drawdowns in the boreholes that intersected the same fracture were very similar in form (except in borehole UO10 in which the fracture does not occur). Because the boreholes used in the test are open and intersect several aquifers, it was not possible to accurately measure the hydraulic gradient. A numerical model was constructed for the site by using the three-dimensional Modflow program (PMWIN, Chiang and Kinzelbach 2001) and using 20 layers of thickness of 0.5 and 1 m, respectively, 10 of which are above and 10 of which are below the main fractured layer (thickness 200 mm). The inverse model PEST (Doherty 2000) was used to calibrate the data and Table 1 shows the results (Van Tonder et al. 2001).

The horizontal K value of the fractured zone was estimated as 3,600 m/day and, if multiplied with the thickness of the fractured zone of about 0.2 m, a T value of 750 m²/day is obtained. Drawdown values obtained from the boreholes after 1.5 min of pumping were used in the Cooper–Jacob distance-drawdown method (see Fig. 6) and a T value of 700 m²/day was estimated, which is very similar to the value estimated with the numerical three-dimensional Modflow model.

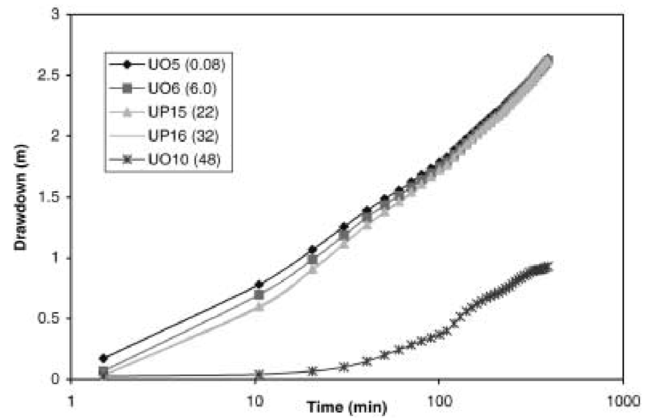

Fig. 5 UO5 pumping test showing the drawdown behaviour in boreholes. UO5 was the abstraction borehole (rate of 1.25 l/s). *Figures* in *parentheses* are distances (m) from the abstraction borehole

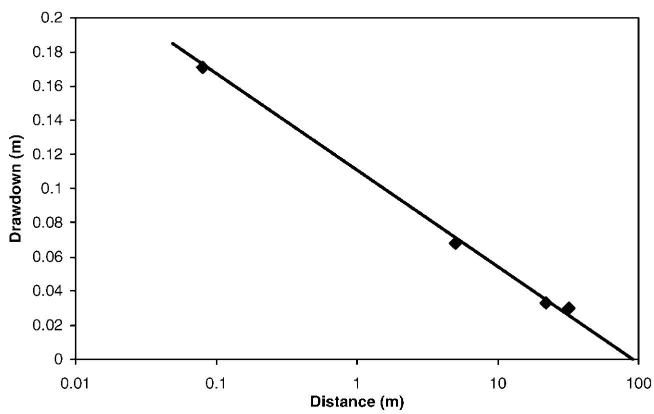

Fig. 6 Cooper–Jacob distance-drawdown method applied to the data of the UO5 test, resulting in a T=700 m²/day for the fracture zone

Table 1 Hydraulic parameters for the aquifer on the Campus Test Site as estimated with the three-dimensional model (K_{hm} horizontal matrix K; K_{vm} vertical matrix K; K_{hf} fracture K; S_{sm} matrix specific storativity)

Parameter	Estimated values
K_{hm} (m/day)	0.158
K_{vm} (m/day)	5.82×10^{-3}
S_{sm} (m^{-1})	5.65×10^{-5}
K_{hf} (m/day)	3.6×10^{3}

The data of the UO5 test was then analysed with the generalized flow model of Barker (1988) and Fig. 7 and Table 2 show the results. For comparison, the fit with a fixed-flow dimension of 2 is included in Fig. 7. From the acoustic scan and the borehole video image, the measured thickness of the fractured zone varied between 0.1–0.2 m. These values were used as lower and upper values for b in the Barker equation [Eq. (1)] to obtain the solution for the other parameters (only the product $K_f b^{(3-n)}$

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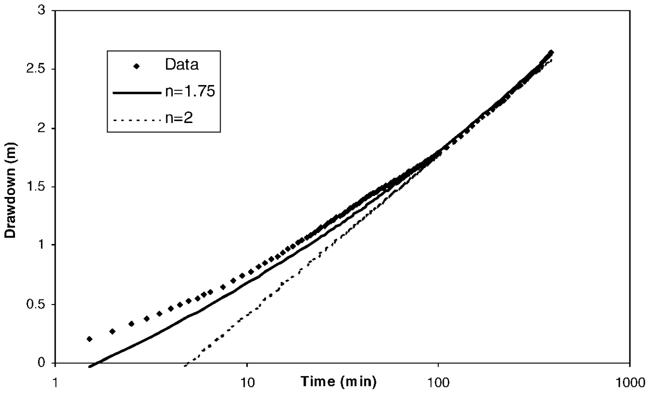

Fig. 7 Barker model applied to the abstraction borehole UO5, yielding a flow dimension of 1.75. For comparison, the simulated drawdown curve with a flow dimension of 2 is included

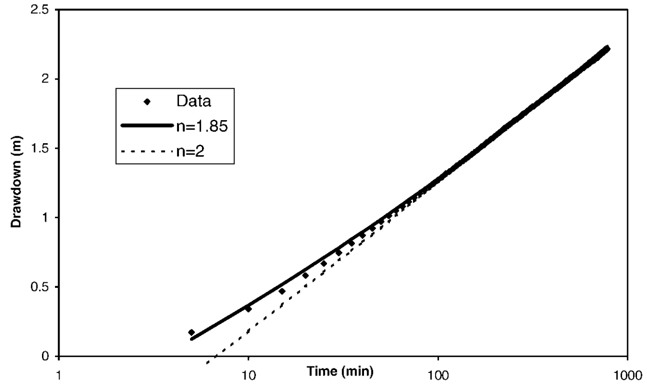

Fig. 9 Barker method applied to the abstraction borehole UO26, yielding a flow dimension of 1.85. For comparison the simulated drawdown curve with a flow dimension of 2 is included

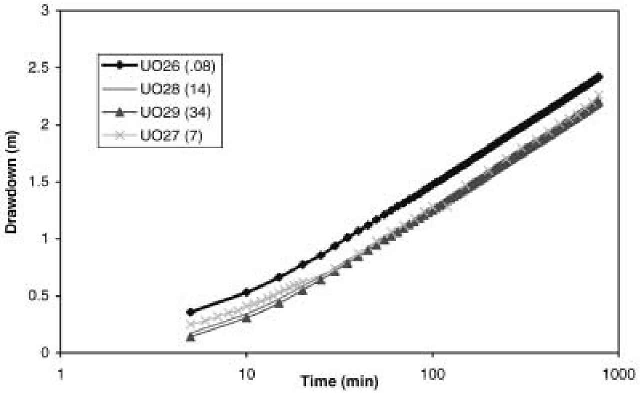

Fig. 8 UO26 pumping test showing the drawdown behaviour in boreholes. UO26 was the abstraction borehole (rate of 0.7 l/s). *Figures* in *parentheses* are distances (m) from the abstraction borehole

Table 2 Parameter values for the UO5 test obtained from the Barker model

Borehole	K_f (m/day)	S_{sf}	b (m)	n
UO5	653	2.6E-3	0.15	1.75
UO6	645	1.1E-3	0.14	1.75
UP15	651	1.5E-3	0.16	1.75
UO16	657	1.7E-3	0.15	1.74

Table 3 Parameter values for the UO26 test obtained from the Barker model

Borehole	K_f (m/day)	S_{sf}	b (m)	n
UO26	199	2.7E-3	0.16	1.85
UO27	187	1.7E-2	0.19	1.80
UO28	201	2.4E-3	0.16	1.84
UO29	200	1.1E-3	0.16	1.85

and diffusivity K_f/S_{sf} can be estimated uniquely with the Barker equation). A non-linear least-square method was applied to estimate the parameters shown in Table 2.

The difference between the estimated K_f value with the Barker equation and that with the numerical model illustrates that there might be a hierarchy of smaller fractures present that possibly forms a fractal system with a lower effective hydraulic conductivity than that of the main fracture.

It can be concluded from the previous analysis that the flow prevailing during the UO5 test is partially dimensioned, called a fractional dimension.

UO26 test

The second test was conducted on borehole UO26, which is situated 15 m from UO5. Water was abstracted at a rate of 0.7 l/s and water-level measurements were taken in boreholes UO27 (distance 7 m), UO28 (distance 14 m) and UO29 (distance 34 m). All the boreholes intersected the same mode I fracture and Fig. 8 shows the pumping-test data in this case. Although borehole UO26 intersected the same bedding-plane fracture as UO5, the pumping-test data show different drawdown behaviour from that obtained during the UO5 test. The drawdown values in UO27, UO28 and UO29 were very similar and did not converge to the same drawdown value as that in the abstraction borehole UO26 (see the pumping-test data for the UO5 test displayed in Fig. 5, where the drawdowns in all boreholes converged to the same value at a late time).

By forcing the b value to be between 0.1–0.2 m, a non-linear least-square fit (see Fig. 9) to the data yield the Barker-model parameter values shown in Table 3. For comparison, Fig. 9 also shows the fit with a fixed-flow dimension of 2.

The flow dimensions n estimated for all boreholes are very similar. Very interesting and puzzling, at the moment, are the high S_{sf} values obtained with the Barker method for both the UO5 and the UO26 tests. The real distances between the abstraction and the observation boreholes were used in the analyses and this could be a

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problem in a practical situation where the actual flow-path length is unknown (called tortuosity). Because of the distance dependency in estimating the storativity, an effective borehole radius of 5 m was used for both the abstraction boreholes to estimate the specific storage, to account for the negative pseudoskin, instead of the real radius of 0.0825 m.

The estimated flow dimensions for the two tests vary between 1.75–1.85, showing a non-integer, fractional-flow dimension. Both the flow dimensions n and the parameter b, estimated with the Barker method, were used to analyse the tracer tests conducted in UO5 and UO26, respectively.

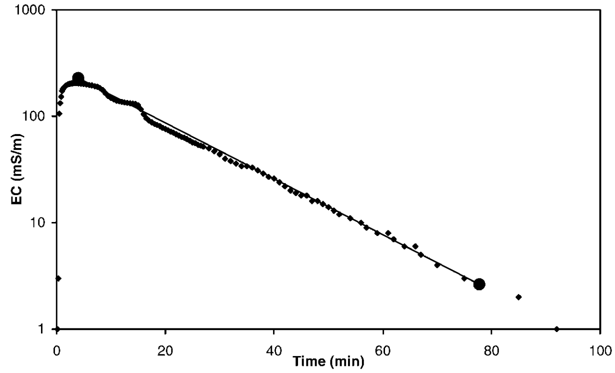

Fig. 10 Point-dilution measurements obtained during the UO5 tracer test in borehole UO20

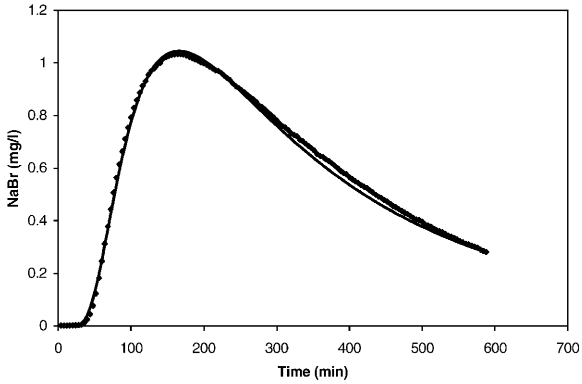

Fig. 11 NaBr breakthrough curve and best fit obtained in the abstraction borehole (UO5) during the radial-convergent UO5 tracer test. Fitted parameters obtained were v=51 m/day, dispersivity of 5.4 m, and thickness of 0.15 m

Tracer Tests

Radial-convergent tests

To estimate the groundwater velocity under forced gradient, the effective thickness of the fracture zone, and the kinematic porosity, two radial-convergent tests were performed, namely (1) between boreholes UO5 and UO20 (15 m apart, called the UO5 tracer test) and (2) between boreholes UO26 and UO28 (14 m apart, called the UO26 tracer test).

UO5 tracer test. Borehole UO20 was used as the source borehole whereas UO5 was used as the abstraction borehole. Water was abstracted from UO5 at a rate of 1 l/s for 3 h (until a steady-state gradient existed between UO5 and UO20). A section in borehole UO20 between 20.5–21.5 m below the surface was sealed off with packers and NaCl and NaBr were introduced into UO20 (mass of 0.048 kg of NaCl and 0.06 kg of NaBr). The solution was mixed by constantly circulating the water with a small pump.

Figures 10 and 11 show the point-dilution and breakthrough curves obtained during the UO5 tracer test. A flow dimension of n=1.75 was used to estimate the through-flow area [Eq. (5)] and this area was used in Eq. (2) to estimate the Darcy velocity as 24.9 m/day. A fit of Eq. (8) to the breakthrough data, obtained in UO5, yielded a seepage velocity of 51 m/day and effective fracture-zone thickness of 0.15 m between UO20 and UO5. The combination of the estimated Darcy and seepage velocity yielded a kinematic porosity value of 0.49. The standard method (i.e. using n=2 and b= thickness of the sealed off section) was also used to estimate the parameters. Table 4 shows the estimated parameter values obtained from the test.

The estimated seepage velocities in Table 4 are both 51 m/day because the same model Eq.(8) was used in both cases. The influence that the flow dimension has on the estimated parameters is revealed by the different values for the Darcy velocity and the kinematic porosities that were obtained.

UO26 tracer test. A radial-convergent test similar to the UO5 tracer test was conducted between UO26 and UO28, with UO28 as the source borehole and UO26 the

Table 4 Parameter values obtained from the UO5 tracer test by using a flow dimension n=1.75 and n=2, respectively. *RCT* Radial-convergent test

Borehole	UO20 (n=1.75)	UO20 (n=2)	UO20 → UO5 (n=1.75)	UO20 → UO5 (n=2)
Darcy velocity (m/day)	24.9	3.48		
Forced-flow velocity (m/day)	51 (from RCT)	51 (from RCT)	51	51
Thickness of fracture zone (m)	0.15 (b from Barker)	1 (sealed-off section)[a]	0.15 (fitted)	0.15 (fitted)
Kinematic porosity	0.49	0.07	0.49	0.12

[a] The length of the sealed-off section was used in this comparison to illustrate the influence of unknown thickness of the fracture zone

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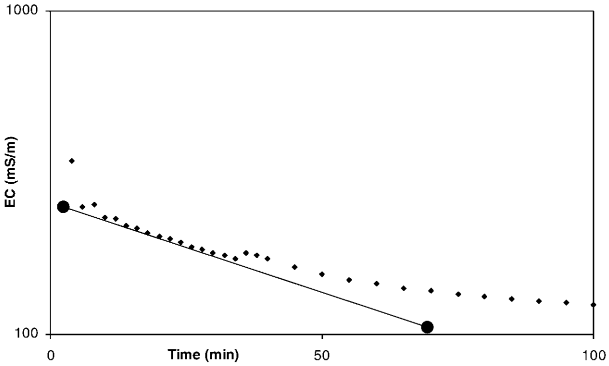

Fig. 12 Point-dilution measurements obtained during the UO26 tracer test in borehole UO28

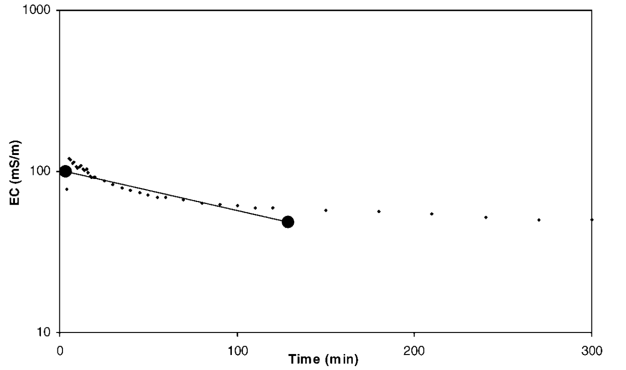

Fig. 14 Point-dilution measurements obtained in borehole UO28 under natural conditions

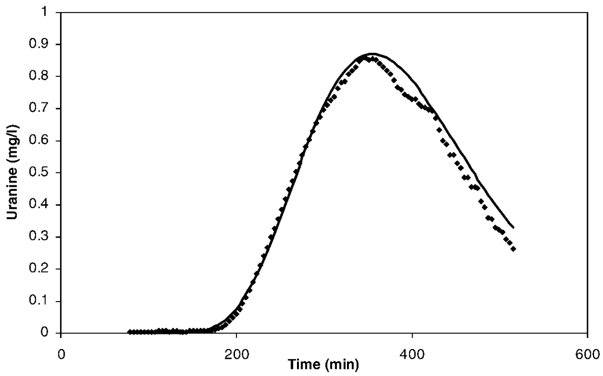

Fig. 13 Uranine breakthrough curve and fit obtained during the UO26 tracer test (radial convergent). Fitted parameters obtained were v=51 m/day, dispersivity is 0.5 m, and thickness is 0.16 m

abstraction borehole. A pseudo-steady-state flow was created by abstracting water from UO26 at a rate of 0.7 l/s for 3 h. A pulse of 0.2 kg NaCl and 0.02 kg uranine was introduced in borehole UO28 in the interval 20–22 m below the surface and mixing of the solution was obtained by constantly circulating the water. A fluorometer was used to measure the uranine breakthrough curve in UO26. Figures 12 and 13 show the point-dilution measurements in UO28 and the breakthrough curve in UO26, respectively. The estimated parameter values are shown in Table 5.

The best fit to Eq. (8) yielded a kinematic porosity of 0.18 and thickness of the fracture zone of 0.16 m. The influence that the flow dimension has on the estimated Darcy velocity and the kinematic porosity clearly can be observed from Table 5.

The boreholes UO20 and UO28 were next used to estimate the natural-flow velocity along the fracture by using the single-well injection-withdrawal test as well as the point-dilution test.

Tracer tests under natural-flow conditions

UO28 natural-gradient tests. A point-dilution test was performed on borehole UO28 by introducing 0.1 kg of NaCl in a section of the borehole from 20 to 22 m. The concentration level was maintained by continuously circulating the water in the 2-m section with a submersible pump at a rate of 0.3 l/s and electric-conductivity measurements were taken with an EC meter. Figure 14 shows the results of the dilution test. Using a flow dimension of 1.85 in Eqs. (2) and (5), the Darcy velocity was estimated to be 4.27 m/day. By using a kinematic porosity of 0.18, as estimated with the radial-convergent test previously described, the natural-flow velocity was estimated as 23.1 m/day.

The point-dilution test was used as the injection part of the injection-withdrawal test. After allowing the tracer to drift away from UO28 for 300 minutes, water was abstracted from UO28 at a rate of 0.3 l/s (see Fig. 15 for the data of the withdrawal phase).

The centre of mass was recovered during the pump-back phase after 60 min (=t_p) and using t_d=301 min and kinematic porosity of 0.18 in Eq. (7), yields a seepage velocity of 21.4 m/day, which is of the same order of

Table 5 Parameter values obtained from the UO26 tracer test by using a flow dimension n=1.85 and n=2, respectively

Borehole	UO28 (n=1.85)	UO28 (n=2)	UO28 → UO26 (n=1.85)	UO28 → UO26 (n=2)
Darcy velocity (m/day)	9.44	0.73		
Forced-flow velocity (m/day)	51 (from RCT)	51 (from RCT)	51	51
Thickness of fracture zone (m)	0.16 (b from Barker)	2 (sealed-off section)[a]	0.16 (fitted)	0.16 (fitted)
Kinematic porosity	0.18	0.02	0.18	0.08

[a] The length of the sealed-off section was used in this comparison to illustrate the influence of unknown thickness of the fracture zone

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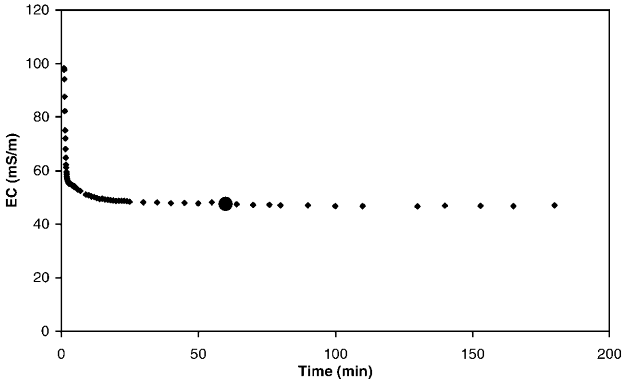

Fig. 15 Tracer measurements made during the pump-back phase of the single-well injection-withdrawal test in borehole UO28. The *large dot* shows the tracer-mass centre

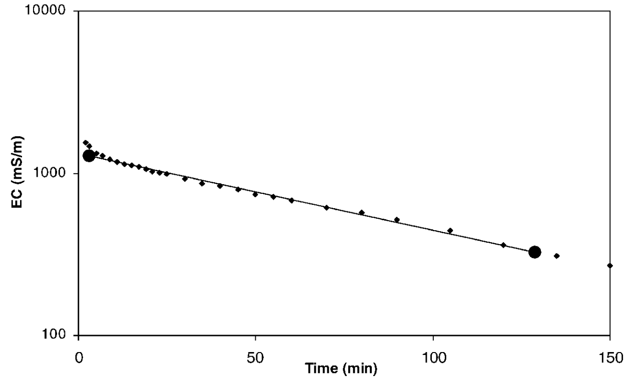

Fig. 16 Point-dilution measurements obtained in borehole UO20 under natural conditions

magnitude as the velocity estimate of 23.1 m/day obtained with the point-dilution test. Both the point-dilution and the single-well injection-withdrawal tests were also analysed using a flow dimension of 2 and the thickness of the section that was sealed off during the test. By using a flow dimension of 1.85 instead of the standard value of 2, the estimated natural velocity is higher by a factor of 2. The parameter values obtained from the tracer tests on borehole UO28 are shown in Table 6.

UO20 natural-gradient tests. A point-dilution test was performed on borehole UO20 by introducing 1 kg of NaCl in a section of the borehole from 20.0–22.5 m. The concentration level was maintained by continuously circulating the water in the 2.5-m section with a submersible pump at a rate of 1 l/s and electrical-conductivity measurements were taken with an EC meter. Figure 16 shows the results of the dilution test.

After allowing the tracer to drift away for 90 min, it was pumped back at a rate of 1 l/s. Figure 17 shows the tracer measurements made among withdrawal phase and

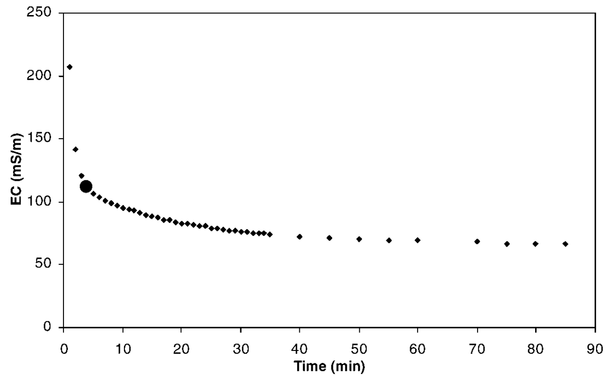

Fig. 17 Tracer measurements made during the pump-back phase of the single-well injection-withdrawal test in borehole UO20. The *large dot* shows the tracer-mass centre

Table 7 shows the resulting parameter values obtained. The estimated average natural-flow velocity by using fractional-flow dimensions is again higher than that obtained with the standard method by a factor of 2.

Table 6 Parameter values obtained from the UO28 point-dilution and single-well injection-withdrawal tests by using a flow dimension n=1.85 and n=2, respectively

	Point-dilution tests (n=1.85)	Point-dilution tests (n=2)	Injection-withdrawal tests (n=1.85)	Injection-withdrawal tests (n=2)
Kinematic porosity (from radial- convergent test)	0.18	0.02	0.18	0.02
Darcy velocity (m/day)	4.27	0.26		
Seepage velocity (m/day)	23.1	12.9	21.4	11.8

Table 7 Parameter values obtained from the UO20 point-dilution and single-well injection-withdrawal tests by using a flow dimension n=1.75 and n=2, respectively

	Point-dilution tests (n=1.75)	Point-dilution tests (n=2)	Injection-withdrawal tests (n=1.75)	Injection-withdrawal tests (n=2)
Kinematic porosity (from radial-convergent test)	0.49	0.07	0.49	0.07
Darcy velocity (m/day)	11.4	0.63		
Seepage velocity (m/day)	23.2	9.28	20.3	10.0

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Summary

The generalized equations (see part 1 of the paper), used to estimate the Darcy velocity (from the point-dilution test) and the flow velocity (from the single-well injection-withdrawal test), were applied to tracer tests conducted at the Campus Test Site in Bloemfontein. It is shown that the velocities estimated with the flow dimension *n* and the extent of the flow region *b*, obtained from the GRF model, are higher than the velocities calculated with the standard method (i.e. flow dimension n=2, b= length sealed-off section of borehole) for the case that the flow dimension *n* is less than 2. For cases where the flow dimension *n* is more than 2 the opposite relation is expected.

Although at the moment it is not possible to directly use the fractional-flow dimension obtained from pumping tests in a numerical-flow model or mass-transport prediction model, the knowledge of the flow dimension is important for the calculation of flow and transport in fractured aquifers. There are several possibilities to account for non-integer-flow dimension in numerical models, e.g. a porous-media model can be used with a spatial variability of stochastically-produced hydraulic parameters, and show the same flow dimension (Van der Voort and van Tonder 2000). However, if the flow dimension that prevails in an aquifer is less than 2 and porous mass-transport models are used to predict contaminant movement, it could easily result in an estimated travel distance that is too small (i.e. the hydraulic conductivity of the fracture zone is underestimated, which will result in an underestimation of the travel distance of the pollutant).

Suggested Procedure for Estimation of Seepage Velocity

1. Conduct a radial-convergent test between two boreholes by using the one borehole as the source for injection and the other as an abstraction borehole.
   - To obtain the flow dimension *n*, and the parameter *b*, which are used to estimate the through-flow area, water-level measurements should be taken during the pumping phase and analysed with the Barker (1988) method. If one uses data from another hydraulic test, the estimated flow dimension could differ from the flow dimension during the tracer test due to the scale effect.
   - Conducting and measuring the injection of the tracer into the source borehole as a point-dilution test and accordingly analysing the test data will give an estimate of the Darcy velocity q_f.
   - Fitting the breakthrough curve measured in the abstraction borehole will yield the groundwater velocity under forced gradient v_f, from which the kinematic porosity could be estimated from the equation $v_f=q_f/\varepsilon$. The kinematic porosity could also be independently estimated from Eq. (16) in Part I of this paper.
2. Perform a single-well injection-withdrawal test on one of the boreholes as follows:
   - Conduct a point-dilution test and allow the tracer to drift away under natural conditions. The Darcy velocity can be estimated from the dilution test using the flow dimension *n* and the parameter *b*, obtained by the Barker (1988) method.
   - Abstract water from the borehole and plot the breakthrough curve from which one can measure the time until the centre of mass of the tracer is recovered. Using the flow dimension *n* and the parameter *b*, obtained from the GRF model, the flow velocity can be estimated from Eq. (10) in Part I of this paper.

Conclusions

The comparison of the estimated parameter values in Tables 4, 5, 6 and 7 shows that the velocities obtained with the equations for fractional dimension are always higher than the values estimated with the standard method for porous media. The estimated values for the flow velocity are sensitive to both the extent of the flow region *b* and the flow dimension *n*. If the thickness of the fracture zone is known by means of borehole logging or can be assumed accurately, it is more appropriate to use this value as parameter *b* instead of the length of the tested section.

Because the flow behaviour of a fractured aquifer can be described with a generalized model like the GRF model from Barker (1988), which accounts for the uncertainty in geometry, it is also correct to implement the fractional-flow dimension for the estimation of transport parameters. Transport and transport parameters depend more on the geometry of the flow system than the hydraulic properties of the aquifer.

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